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by

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13. ABSTRACT

This paper develops and illustrates an approach for analytically assessing the impacts on both costs and service of consolidation of repair facilities. The repair facilities are two echelon generalizations of the classical repairmen problem in which two types of failures occur, requiring repair at different echelons. The types of questions raised include the reduction in space possible under the consolidated configuration and yet still provide the same level of service, and the physical separation between the users and the consolidated repair facility that is economical. The method of analysis is based upon asymptotic approximation developed for the repairmen problem, valid when the number of operational equipments is large (greater than 50). Three traffic intensity situations are investigated, depending upon the relationships of the number of spares, number of service channels, failure rates and repair rates.

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1.0 INTRODUCTION AND SUMMARY

Recently, due to the austere funding levels facing the Navy, it has become important to be able to investigate the impacts, on both costs and service levels, of consolidation or centralization of repair activities. In particular, it is of interest to consider a consolidated system in which all the resources (spares, repairmen, repair parts, etc.) previously owned and managed by several disjointed facilities with no mechanisms for sharing are combined in one facility. Such a consolidation prevents imbalances from occurring where, for example, idle repair capability could exist at one facility while at the same time excessive delays could be occurring at other facilities.

Among the key questions to be raised are:

1) Given the consolidated facility has the same resources as available under the disjointed configuration, what is the increased level of service (in terms, for example, of meeting some minimum operational requirements) that can be provided.

2) What reduction in spares, repairmen or repair rates can be tolerated under the consolidated arrangement and still provide the same level of service?

3) Since it is clear that one of the disadvantages of consolidation might be the longer turn-around time (due to the decreased proximity in some cases of the repair facilities to the users), what kind of

separation between the users and the consolidated facility
is practical?

With these questions as motivation, this paper develops a tractable, analytical approach for assessing the impacts of consolidating several disjointed repair facilities. The repair facilities considered in this paper are two-echelon generalizations of the so-called classical repairmen problem (for example, see The Mathematical Theory of Reliability, Richard E. Barlow and Frank Proschan, Wiley & Sons, 1965) for which it is assumed:

1) There is an operational requirement for N equipment to be functioning continuously (otherwise the system's performance is degraded). In addition the equipments are all assumed to be stochastically independent of one another and fail according to some specified distribution with a mean time between failure of $1/\lambda$.

2) Backing up these N operational equipments are M spares, which can be used to fill any of the operating slots on an as needed basis.

3) Two types of failure are considered, called major and minor, (in general the approach can be easily extended to cover an arbitrary number of failure types). With probability p, a failure results in a requirement for service from Repair Echelon 1 (perhaps a local repair facility or tender); with probability $1-p$ the failure

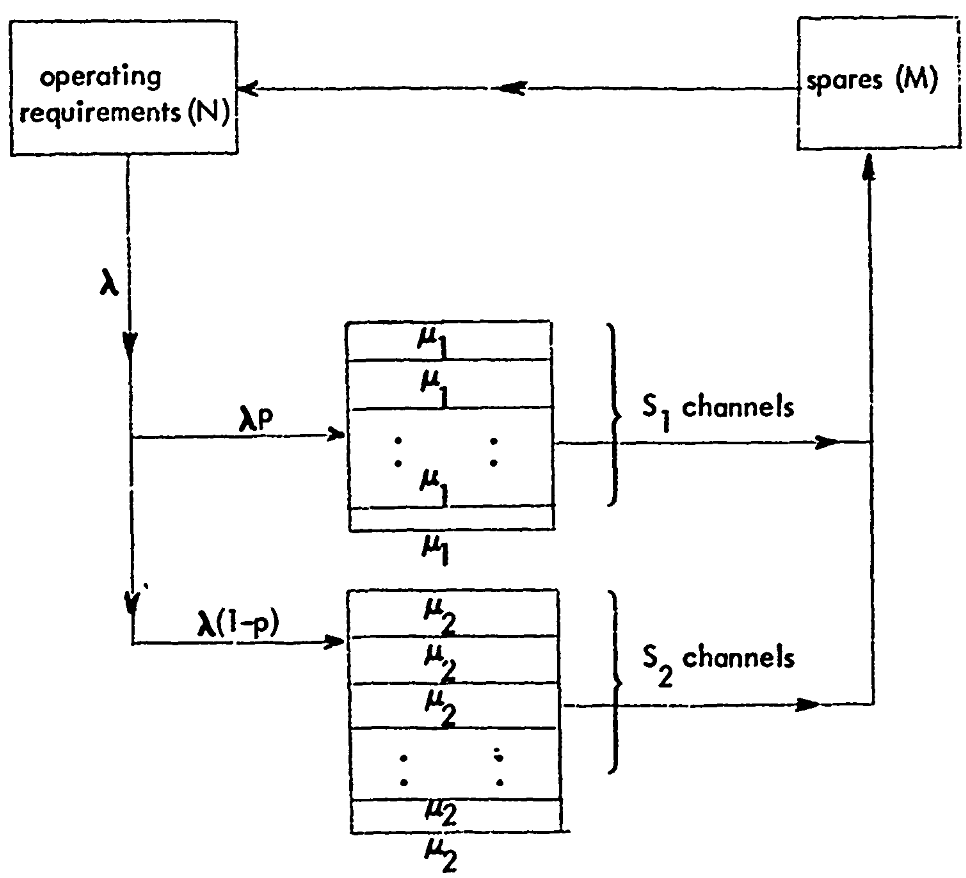
is more serious and can be repaired* only at Repair Echelon 2.

4) Both repair facilities are capable of repairing only a fixed number of units simultaneously, S_1 for Echelon 1 and S_2 for Echelon 2. If all repairmen are busy at a given Echelon, the failed item joins the waiting line at the Echelon and waits until a repairman is free. In addition the repair times at each Echelon are also independent, identically distributed random variables with means $1/\mu_1$ for Echelon 1 and $1/\mu_2$ for Echelon 2.

5) Finally, when repairs are completed, they return to the spare pool and are once again available to fill operational openings.

The flow is as depicted below.

Figure 1



* An alternate interpretation of the repair activity at Repair Echelon 2 could be a one-for-one ordering. This would be appropriate if the major failure were catastrophic.

The method of analysis utilized in this paper is based upon an application of asymptotic approximations developed in a previous Control Analysis Corporation Report (see, "Approximations for the Repair Problem with Two Repair Facilities, II: Spares", CAC Technical Report 266-4, Iglehart and Lemoine, Ocotber 1972). The approximations, appropriate when the number of operating equipment (denoted N earlier) is large*, are valuable in that they provide a tractable means for predicting the steady-state system performance as a function of the parameters $N, M, S_1, S_2, \lambda, \mu_1, \mu_2$ and p. Hence, such questions as the reduction in spares that can be tolerated in the consolidated scheme and still yield the same system performance as in the disjointed arrangement can be answered readily without resorting to an exhaustive computation of all the exact steady state probabilities. The approximations utilized can be divided into three classes based upon the level of the so-called traffic intensity or amount of congestion expected. The classes, referred to as light, intermediate and heavy, refer to the relationships between the number of spares, service channels, arrival rates and service rates. The actual approximations utilized are presented in the Appendix.

Several numerical examples are presented in Section 2, illustrating the approach for each of the light, intermediate and heavy traffic intensity cases. It should also be stressed that the results available at this time require a Poisson failure process and exponential repair times. Although the first assumption is quite reasonable, the exponential repair assumption is quite severe in that realistic repair distribution

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* Calibration studies have shown that for N's in excess of 50, the approximations are quite accurate, i.e. within 2-3%.

usually follow a lognormal or Weibull distribution. Hence, further efforts in this area should be geared to relaxing this assumption. In addition, for ease of presentation due to the number of parameters involved, it has been assumed each of the disjointed facilities are identical. In practice, it would be no problem to consider the consolidation of non-identical facilities. Also each of the examples presented consider the implication of consolidating two disjointed facilities; again the approach is capable of assessing the impact of consolidation of any number of facilities.

2.0 NUMERICAL RESULTS

Example 1: Light Traffic Intens ty Repair, Two Echelons

Consider the following set of parameters, applicable for each of the two disjointed facilities:

N	the number of operational slots, equals 75
M	the number of spares, equals 13
$1/\lambda$	the mean time between equipment failures, equals $6\frac{2}{3}$ days
P	the probability of a failure being minor, equals .9
1-p	the probability of a failure being major, equals .1
S_1	the number of service channels at Echelon 1, equals 6
S_2	the number of service channels at Echelon 2, equals 4
$1/\mu_1$	mean repair time (not including waiting time) at Echelon 1, equals $\frac{1}{2}$ day
$1/\mu_2$	mean repair time at Echelon 2, equals 2 days.

Then, it can be shown using the formulae in the Appendix that since $S_1+S_2 \leq M$, $\frac{\lambda N p}{S_1 \mu_1} < 1$, and $\frac{\lambda(1-p)}{\mu_2 S_2} N < 1$ a light traffic intensity situation occurs with the result that:

1) the likelihood that at each of the two disjointed activities the 75 operational slots are filled (or equivalently that between the two echelons there are less than or equal to 13 equipments being repaired or awaiting repair) equals .91. Hence, the probability that this is so for two disjointed activities is $(.91)^2$ or .83.

2) In comparison, if the two disjointed facilities, with no sharing of operational units, spares, or repairmen, are replaced by one facility having twice the resources available to it, namely 26 spare equipments, 12 service channels capable of repairing minor failures, and 8 service channels capable of repairing major failures, and further if the consolidated facilities minimum operating requirements are the sum of the two disjointed requirements, namely 150 equipments, then the probability of these operational requirements being met (or equivalently the probability there are less than or equal to 26 units in the repair cycle) can be shown to be .998. Hence, for the same resources, disregarding any increased transportation expenses, there is a substantial gain (namely 81% to 99%) in the level of service provided.

3) Under the consolidated arrangement, a reduction of 4 spares (i.e., instead of the 26 spares used in the decentralized arrangement, only 22 are required in the consolidated scheme) or a 17% reduction, can be tolerated and still yield the same level of protection as in the decentralized scheme. If only 95% of the operational slots need to be filled, then a 25% reduction in spares can be achieved under the consolidated arrangement and still deliver the same level of protection as provided under the decentralized scheme.

4) Under the consolidated arrangement either a 34% degradation in the service rate at the first echelon (i.e. $1/\mu_1$ can increase from $\frac{1}{2}$ day to .68 days), or a 79% degradation of the service rate at the second echelon (i.e. $1/\mu_2$ can increase from 2 days to 3.58 days)

can be tolerated and still yield the same level of protection as in the decentralized scheme.

5) Under the consolidated arrangement, 4 minor repair service channels and 2 major repair channels can be removed and still meet the level of service provided under the disjointed scheme.

Example 2: The Tradeoff Between Efficiency and Proximity

The following example is presented to illustrate the tradeoff between the reduction in spares obtainable as a result of the consolidated system's increased efficiency versus the increased inventory of equipments needed in the pipeline. The additional pipeline inventory is required in the consolidated system since under this scheme the decentralized, independent facilities may be located close to the users. However, in the consolidated scheme, it will not be possible for the central facility to be close to all users, and hence the need for more pipeline inventory.

To concretely illustrate the tradeoff involved, consider the following set of parameters for the one repair echelon case:

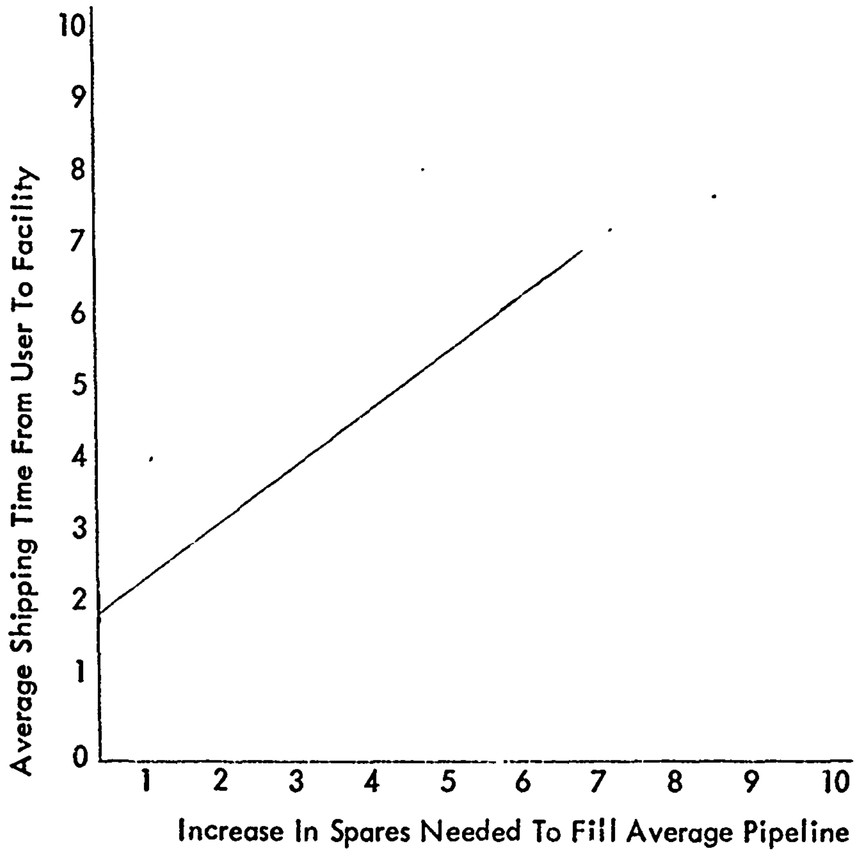
N	the number of operational units, for each of two independent facilities is 50
M	the number of spares at each is 10
S	the number of service channels at each is 7
$1/\lambda$	the mean time between failure is 100 days
$1/\mu$	the mean repair time is 10 days

Finally, suppose that under the disjointed arrangement, each facility serves a different set of users and that the large distance between the two facilities precludes any sharing of operational units, spares or repairmen.

Then, in this setting, consider replacing the two independent facilities by one large facility which is located midway between the previous two facilities, such that the one-way increased shipping time from the users to the repair facility is, say T days. Then the question to be answered is, "As a function of T, what is the increase, if any, in the total number of spares required under the consolidated scheme to fill the increased average pipeline requirements due to the additional T shipping days from the users to the facility?"

Figure 2, derived in the Appendix, answers this question. The case considered assumes that it is desired to be able to maintain or fill a total of 2N or 100 operational slots. Note that in this case the increased efficiency afforded by the consolidation permits a separation of about 1.9 days between the users and the consolidated facility without any increase in the number of spares; also if T were increased to about 5 days, an additional 6.2 spares would be required to fill the additional average pipeline.

FIGURE 2
THE CONSOLIDATED SYSTEM'S ADDITIONAL SPARE REQUIREMENTS
AS A FUNCTION OF THE DISTANCE FROM USERS
TO THE CONSOLIDATED REPAIR FACILITY



The above graph depicted the additional spare requirements, assuming that it was important to maintain 2N operational slots. The following table is presented to show the maximum additional spacing between the users and consolidated facility that can be tolerated, with no additional spares required, as a function of the fraction of operating slots it is desired to be able to fill. The spacing in this case is determined such that the reduction in spares, brought about by the gain in efficiency under consolidation, offsets the average inventory required to fill the additional pipeline time under the consolidated scheme.

Service Level*	Probability Of Achieving Service Level	Maximum Separation Between Users and Consolidated Facility With No Additional Spares Available
1.00	.86	1.91
.99	.88	2.03
.95	.95	2.63
.90	.98	4.80
.80	.99	9.78

* Fraction of the 2N operational slots required to be filled.

Example 3: Heavy Traffic Intensity

The previous two examples presented were characterized by both having so-called light traffic intensity, i.e. roughly speaking, the rate at which equipments were breaking down is less than the rate at which they could be repaired. However, note that since the problem being studies is a closed system, i.e. no new equipments are entering or escaping the system, it makes sense to consider situations in which the above does not hold. In this vein, this example and the following example illustrate the types of results available under these not so high traffic intensity conditions. In particular the case considered here is a so-called heavy traffic case in which $N\lambda/S\mu > 1$. In this case it is shown that additional spares are of no help in improving the system's performance. To illustrate the system's performance, consider two disjointed systems, each of which can be characterized as follows:

N	the number of operational slots is 75
M	the number of spares is 13
S	the number of service channels is 10
λ	the breakdown rate is .15
$1/\mu$	the mean repair time is .975

Then, it can be shown, (see Iglehart and Lemoine) that at each separate facility all the service channels will be occupied and queues of the order of $N+M-S(\mu/\lambda+1)$ or about 8 will form. In addition, roughly $S\frac{\mu}{\lambda}$ or 71 units will be operating, regardless of the level of spares. Using the formulae prescribed in the Appendix, the following results are obtainable for use in comparing the system performance of the two disjointed activities with that of the single consolidated activity facility having $2N$ operational units, $2M$ spares, and $2S$ service channels:

Service Level Desired (fraction of the 150 operational slots required by be filled)	Likelihood Of Achieving Service Level Under Unconsolidated Arrangement	Likelihood Of Achieving Service Level Under Consolidated Arrangement with Identical Resources	Percent Decrease In Average Service Time Possible Under Consolidated Scheme with No Reduction In Service
100%	2%	8%	4%
95%	8%	23%	5%
90%	21%	44%	6%
75%	81%	96%	9%
50%	99%	99+%	23%

Example 4: Intermediate Traffic Intensity

This example deals with a situation which perhaps is not too realistic, i.e. the number of spares is less than the number of service channels. In particular, it requires $M < N\lambda/\mu < S$, and in fact that the number of spares not be "too close" to $N\lambda/\mu$ (see the Appendix for details). In this case it can also be shown that with high probability there will be idle repair capability but at the same time some of the N operational slots will not be able to be filled. In this situation, in contrast to the previous one, it pays to add spares. Consider the situation of Example 4, where, instead of 13 spares and 10 service channels, there are 8 spares and 13 service channels. Then the following results are derivable, using the methodology presented in the Appendix, for comparing the consolidated system's performance with that of the decentralized system.

Service Level Desired (fraction of the 150 operational slots required to be filled)	Likelihood Of Unconsolidated System Achieving Stated Service Level	Likelihood Of Consolidated System With Identical Resources Achieving Stated Service Level	Percent Reduction In Spares Required Under Consolidation Scheme To Achieve Stated Service Level	Percent Decrease In Average Service Time Under Consolidated Scheme To Achieve Stated Service Level
100%	3	11	16	16
95%	38	70	18	17
90%	88	98	46	22

APPENDIX: METHODOLOGY FOR ESTIMATING THE IMPACTS OF CONSOLIDATION

This Appendix presents the formulae used to compute the results of the numerical illustrations of Section 2. The methodology utilizes the asymptotic approximations developed by Iglehart and Lemoine for estimating the operating characteristics of repairmen problems when the number of operational units N is large. The steady state approximations presented, one each for the cases of light, intermediate and heavy traffic intensity, utilize the normal distributions. They express the statistics of the number of units awaiting or being repaired as a function of the many parameters involved. Hence, since the system is a closed one, it is an easy matter to determine the likelihood, for a given facility, of having various fractions of the operational slots filled. In particular, if X denotes the number of units awaiting or undergoing repair, the number of operational slots filled is $N-(X-M)^+$. The impact of consolidating such facilities can be straightforwardly approximated using the same approach but with twice the number of units and twice the resources. This is then compared with the service levels achieved jointly by the two independent facilities, where it is assumed that because of their separation they are not able to share operational units, spares, or repair capabilities. The three approximations utilized (their proofs to be found in CAC Technical Report 266-4, "Approximations for the Repairman Problem with Two Repair Facilities, II: Spares", Iglehart and Lemoine, October 1972) are:

Case I: Two Echelons, Light Traffic

Let N be the number of operational units, $S_i (i=1,2)$ denote the number of service channels at echelon i, M denote the number of spares, λ denote the equipment failure rate, μ_i the repair rate at echelon i, and p the probability that a failure requires repair at echelon 1. Further, let $X_i (i=1,2)$ denote the steady state number of units awaiting and undergoing repair at echelon i. Then it can be shown, since the process (X_1, X_2) is a positive recurrent Markov Chain with finite state spaces $\{(i,j) : i,j \geq 0;\ i+j \leq N+M)\}$, that if:

$$S_1+S_2 \leq M \quad \text{and} \quad \frac{\lambda p N}{\mu_1 S_1} < 1 \quad \text{and} \quad \frac{\lambda(1-p) N}{\mu_2 S_2} < 1$$

then for large N, X_1 and X_2 are independent, normally distributed random variables with means

$$\frac{\lambda p}{\mu_1} \quad \text{and} \quad \frac{\lambda(1-p)}{\mu_2}$$

respectively, and variances

$$\frac{\lambda p}{\mu_1} \quad \text{and} \quad \frac{\lambda(1-p)}{\mu_2}$$

respectively.

Case II: One Echelon, Heavy Traffic

Using the notation as in Case I, suppose

$$\frac{N\lambda}{S\mu} > 1 \Big/ \left[1-\left(\frac{1}{N}(S-M)^{+}\right)\right]$$

then X, the steady state number awaiting or undergoing repair is normally distributed with a mean of $N+M-S\mu/\lambda$ and variance $S\mu/\lambda$.

Case III: One Echelon, Intermediate Traffic

Using the notation as above, suppose

$$\frac{N\lambda}{M\mu} > 1, \quad \frac{N\lambda}{S\mu} < 1, \quad \text{and} \quad \left[e\,(\lambda/\mu)(1+M/N) \Big/ M/N(1+\lambda/\mu)\right]^{M/N} (1+\lambda/\mu)^{-1} = \gamma < 1$$

then X, the steady state number awaiting or undergoing repair is normally distributed with a mean of

$$\left(N\frac{\lambda}{\mu} + M\right) \Big/ \left(1+\frac{\lambda}{\mu}\right)$$

and variance

$$(N+M)\frac{\lambda}{\mu} \Big/ \left(1+\frac{\lambda}{\mu}\right)^2$$

Finally, the procedure for determining the increased average pipeline inventory required under the consolidated scheme is an application of Palm's theorem which states that if the demand is Poisson with parameters α and the average pipeline time is $\bar{L}$, then the average pipeline inventory is simply $\alpha\bar{L}$. For the illustration of Example 2, a situation of light traffic existed in which, with very high probability, all N operational slots were filled. Hence, the rate at which equipment are breaking down in this case is approximately $N\lambda$. In addition, suppose the consolidated facility is

located midway between two groups of users, such that the additional shipping time, say from the user to the repair facility, is Δ units of time. Then the additional total average pipeline inventory required to take into account this increased shipping time is $4N\lambda\Delta$. This follows since for both sets of users, the failed items must be shipped from the user's area to the repair facility and then the repaired item is returned to the user.

REFERENCES

1 The Mathematical Theory of Reliability, Richard E. Barlow and Frank Proschan, Wiley and Sons, 1965.

2 "Approximations for the Repair Problem with Two Repair Facilities, II: Spares", Control Analysis Corporation Technical Report 266-4, D.L. Iglehart and A. J. Lemoine, October, 1972.